Binomial Theorem Worksheet

1. Expand $(x+2)^5$ using the binomial theorem.
2. Find the coefficient of x^3 in the expansion of $(2x-1)^6$.
3. Using the binomial theorem, prove that:
$$\sum_{k=0}^{n} nk = 2^n$$
4. Determine the middle term(s) in the expansion of $(x+y)^{10}$.
5. If $(1+x)^n = 1+10x+45x^2+\ldots$, find the value of n.
6. Show that $nk = n-1k-1+n-1k$.
7. Compute the sum:
$$\sum_{k=0}^{4} 6k$$
8. Write the general term of the expansion of $(2x-3)^8$ and simplify it.
9. Find the term independent of x in the expansion of $\left(x^2+\frac{1}{x}\right)^6$.
10. Using the binomial theorem, evaluate:
$$(1+0.2)^5$$
11. Prove that:
$$nr + nr + 1 = n + 1r + 1$$
12. Expand $\left(1-\frac{2}{x}\right)^4$ up to three terms.
13. Solve for k if $10k = 10k+2$.
14. Evaluate the sum:
$$\sum_{k=0}^{n} k \cdot nk$$
15. Find the coefficient of x^4y^6 in the expansion of $(x+y)^{10}$.

Solutions to Binomial Theorem Worksheet

1. **Solution to Question 1:** Expand $(x+2)^5$ as:

$$(x+2)^5 = \sum_{k=0}^{5} 5k x^{5-k} 2^k$$

Explicitly write each term:

$$(x+2)^5 = x^5 + 10x^4(2) + 10x^3(2^2) + 5x^2(2^3)$$

$$+1x(2^4) + 2^5$$

Simplify:

$$(x+2)^5 = x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$$

2. **Solution to Question 2:** The coefficient of x^3 in $(2x-1)^6$ is given by:

$$63 \cdot (2)^3 \cdot (-1)^3$$

Calculate each term:

$$63 = 20, \quad (2)^3 = 8, \quad (-1)^3 = -1$$

Coefficient:

$$20 \cdot 8 \cdot (-1) = -160$$

3. **Solution to Question 3:** Use the binomial theorem:

$$(1+1)^n = \sum_{k=0}^{n} nk$$

Simplify:

$$2^n = \sum_{k=0}^{n} nk$$

4. **Solution to Question 4:** The middle term(s) in the expansion of $(x+y)^{10}$ is given by:

$$10k x^{10-k} y^k$$

For $k = 5$:

$$105x^5y^5 = 252x^5y^5$$

5. **Solution to Question 5:** Given $(1+x)^n = 1 + 10x + 45x^2 + \ldots$, compare coefficients:

$$n1 = 10 \quad \Rightarrow \quad n = 10$$

6. **Solution to Question 6:** Use Pascal's identity:

$$nk = n - 1k - 1 + n - 1k$$

7. **Solution to Question 7:** Compute:

$$\sum_{k=0}^{4} 6k$$

Using the binomial theorem:

$$\sum_{k=0}^{6} 6k = 2^6 = 64$$

For $k = 0, 1, 2, 3, 4$, add:

$$60 + 61 + 62 + 63 + 64$$

$$= 1 + 6 + 15 + 20 + 15 = 57$$

8. **Solution to Question 8:** Write the general term of $(2x - 3)^8$:

$$T_k = 8k(2x)^{8-k}(-3)^k$$

Simplify:

$$T_k = 8k2^{8-k}x^{8-k}(-3)^k$$

9. **Solution to Question 9:** Find the term independent of x in $\left(x^2 + \frac{1}{x}\right)^6$:

$$T_k = 6k(x^2)^{6-k}\left(\frac{1}{x}\right)^k$$

Solve for k such that the power of x becomes zero:

$$2(6-k) - k = 0 \quad \Rightarrow \quad k = 4$$

Term:

$$T_4 = 64(x^2)^2\left(\frac{1}{x}\right)^4$$

$$= 64x^4 \cdot x^{-4} = 64$$

$$T_4 = 15$$

10. **Solution to Question 10:** Using the binomial theorem:

$$(1 + 0.2)^5 = \sum_{k=0}^{5} 5k(0.2)^k$$

Approximate:

$$= 1 + 5(0.2) + 10(0.2)^2 + 10(0.2)^3$$

$$+5(0.2)^4 + (0.2)^5$$

Simplify step-by-step.

11. **Solution to Question 11:** Prove:

$$nr + nr + 1 = n + 1r + 1$$

Use Pascals identity:

$$nr + nr + 1 = n + 1r + 1$$

12. **Solution to Question 12:** Expand $\left(1 - \frac{2}{x}\right)^4$ up to three terms:

$$\left(1 - \frac{2}{x}\right)^4 = 1 - 4\left(\frac{2}{x}\right)$$

$$+6\left(\frac{2}{x}\right)^2 + \dots$$

Simplify:

$$= 1 - \frac{8}{x} + \frac{24}{x^2} + \dots$$

13. **Solution to Question 13:** Solve for k in $10k = 10k + 2$:

$$10k = 1010 - k - 2$$

$$k = 4$$

14. **Solution to Question 14:** Evaluate:

$$\sum_{k=0}^{n} k \cdot nk = n \cdot 2^{n-1}$$

15. **Solution to Question 15:** Coefficient of x^4y^6 in $(x+y)^{10}$:

$$104x^4y^6$$

Coefficient:

$$104 = 210$$